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OBSERVATION OF THE GOLDEN RATIO IN SPIRALS OF TRIANGLES AND SQUARES

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Abstract. In this paper we introduce the notion of *L golden gnomon*, and some properties of it are observed as applied to the growth of spirals. Spirals of squares and of triangles are especially observed, with relevance of the *golden ratio* as a geometrical link between the two polygons.

Key words. Golden ratio, gnomon, spiral, square, triangle.

Mathematics Subject Classification: 51M15

Introduction and definitions

In this paper we use the notation $\varphi = \sqrt{(5/4)} - (1/2)$ and $\Phi = \sqrt{(5/4)} + (1/2)$ for the *golden ratio*. The present observations develop some aspects of the main geometrical discovery of the named *L golden gnomon* [1].
This gnomon is equivalent to the growth of one-quarter of the surface of a square: the *golden ratio* is contained in this simplest geometrical operation on the square (see figure 2).

Fig. 1 shows the classic construction [2] of the *golden section* of the side of a square by reporting the segment OA in OC, where BC is the φ value of AB.

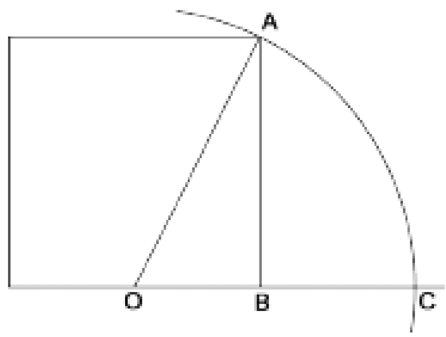


Fig. 1. Euclidean construction of the *golden section* of the side of a square

1. The *L golden gnomon*

In Fig. 2, an equivalent segment is reported in the other direction, so defining the measure of a gnomon named *L golden gnomon*:

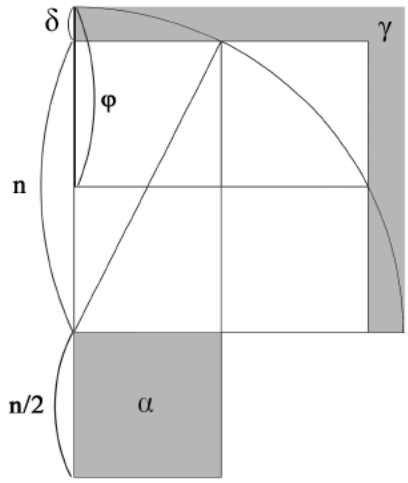


Fig. 2. Construction of the *L golden gnomon*

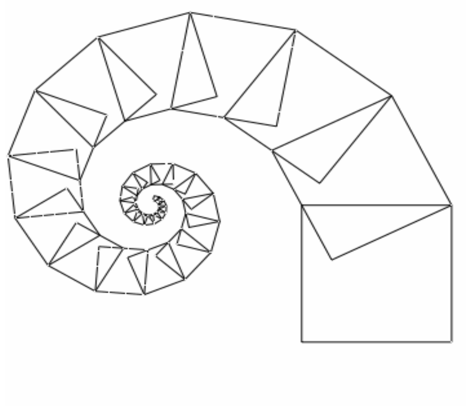
The area of the *L golden gnomon* is equivalent to 1/4 of the square, so that $\alpha = \gamma$.

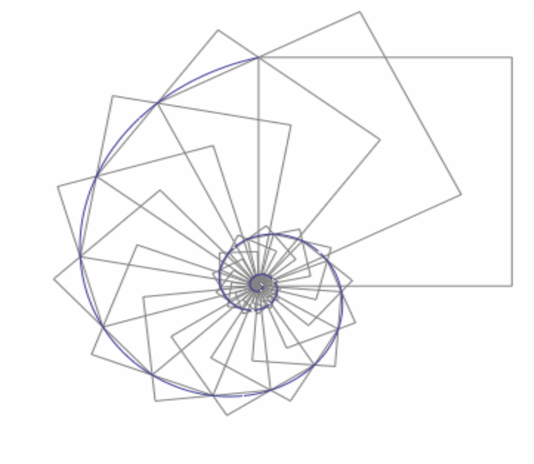
This can also be expressed in a formula:

$$\sqrt{(n^2 + (n/2)^2)} + (n/2) = \Phi n \quad \text{and} \quad \sqrt{(n^2 + (n/2)^2)} - (n/2) = \varphi n \tag{1}$$

2. Spirals of squares

A family of logarithmic spirals can be sorted out by the geometrical application of **(1)**.
The winding of the spirals is determined by the *L golden gnomon* that makes any successive square growing (or decreasing) one-quarter bigger (or smaller). As it is shown by **(1)**, the *golden ratio* is implicit between squares in proportion 4/4 and 5/4.
The direction of growth of the square by *L golden gnomon*, also determines the different kind of the spiral.





Another interesting shape generated by the successive growth of one-quarter of square is shown in Fig. 4, where the successive *L golden gnomons* are translated and rotated by 90° clockwise, generating a double spiral:

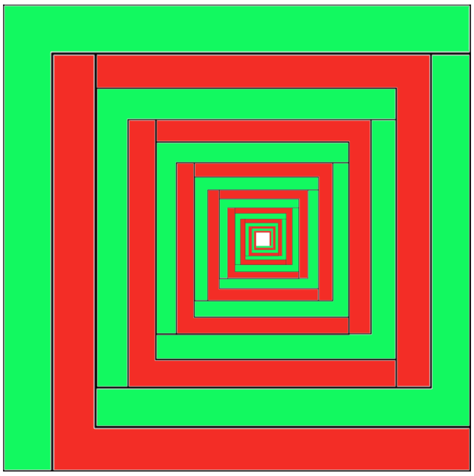


Fig. 4. Two interlocked spirals

Fig. 5 shows the linear growth by *L golden gnomons* of the sides of successive squares.
As this is ruled by the ratio of √(5/4) it is easily demonstrable that, considering the sides of three successive squares, the sum of the linear measure of two successive *L golden gnomons* equals to 1/4 of the side of the first square, and 1/5 of the third one, that is:
BC + CD = 1/4 AB = 1/5 AD.

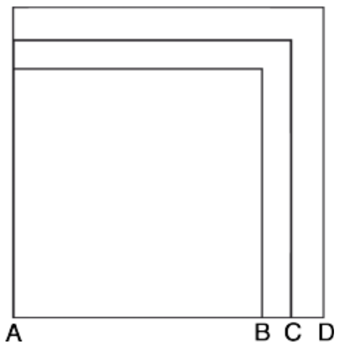


Fig. 5. Three successive squares with growth of 1/4

3. Spiral of triangles

The same basic rule used in Fig. 1 for the growth of the square can be generalized and applied to the triangle: *be the mid-point of one side of the polygon joint with the opposite vertex*.
A spiral of triangles can be produced as in Fig. 6, where the side of every triangle is naturally equal to the height of the previous one.

The side of the bigger triangle is the diameter of the circle inscribing the smaller one, so that the areas grow of one-third.

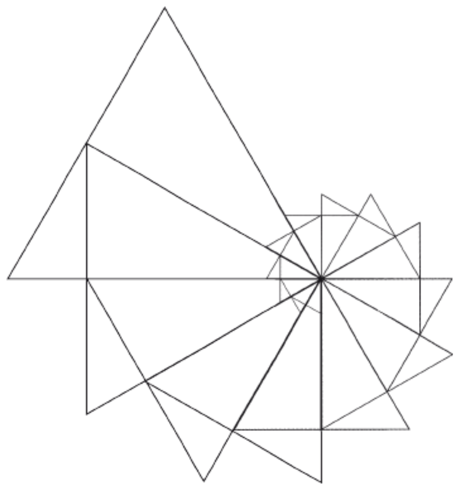


Fig. 6. Growth of 1/3 in a spiral of triangles

It is now possible to observe the *golden ratio* in the growth of the triangle of one-third, finding also a connection to the growth of the square of one-quarter.

In Fig. 7, be CE = 1, then AE = $\sqrt{(5/4)} + (1/2) = \Phi$ [3].

It is intuitive to demonstrate with the general properties of the triangle and the theorem of Pythagoras, that AB = $\sqrt{(5/4)} - 1$.
Then AS = $\varphi\sqrt{2}$ and AF = $\sqrt{2}$.
By the theorem of Thales on proportional line segments, it is demonstrated that point S cuts in *golden section* also segment EF[1], so that ES = $\varphi\sqrt{3}$.

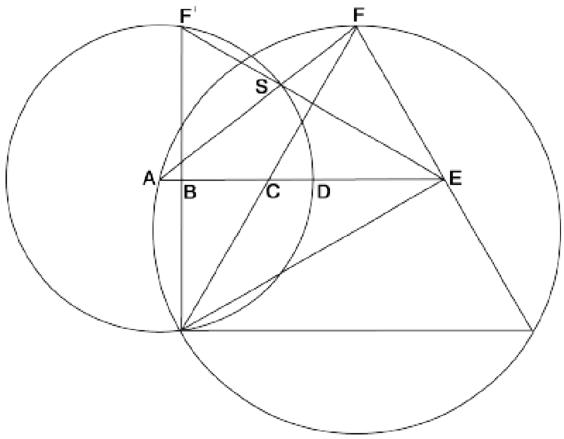


Fig. 7. Growth of one-third in the triangle with *golden ratio* of √2 and √3

The link between triangle and square by *golden ratio* is here expressed by the unexpected (and beautiful) presence of the segment measuring √2 [4].

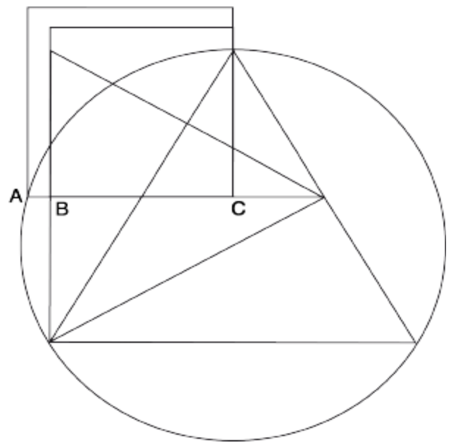


Fig. 8. *L golden gnomon* constructed on the triangles

In Fig. 8 it is now constructed the square with side BC = 1, then AB will be the measure of a *L golden gnomon* with area 1/4.

4. Conclusion

Studies on the *golden* properties in the triangle have been produced only in recent times, compared to the official history of the *golden ratio*.
This indicates anomalies in this field of the research on *golden ratio* where a general objective vision is still missing, and open is the debate on the presumed "beauty" of the "divine proportion" [5] and its importance in the rules of nature.
In the present paper we presented interesting new approaches to lost ideas [6], such as the construction of the *L golden gnomon*.
We used the word "beautiful" once in the above lines, referring to the mathematical surprise of seeing the diagonal (√2) of the unity square coming out of *golden* properties of the triangle. It is a mathematical shock, opening the way to the contemplation of beautiful images that can be derived from this geometrical unobserved properties.
This is not a definitive answer to what is "beauty", of course; the aim of this conclusion is to focus and define the language for talking about it with objective ideas and creative method.

References

[1] The genesis of this idea is described in the book Gelatti, *Il quadrato di 9 e la sezione* aurea, Sagep, 2011, and in the paper *The golden gnomon and the Phi quadratic formula, with geometrical problems in Fibonacci-Lucas Numbers* presently submitted to the Aplimat journal 2014.
[2] Euclid, Elements II, 11.
[3] Odom, *Problem E3007*, *American Mathematical Monthly*, 1983.

[4] See also Bataille, *Another Simple Construction of the Golden Section*, Forum Geometricorum Vol. 11 55, 2011; see also Gelatti, *La sezione aurea nel triangolo equilatero: psichismo ed esoterismo dei mandala e degli yantra*, under publishing in the Proceedings of the International AISPES Conference *Il numero d'oro*, Genoa University, November 2012.
[5] Pacioli, *De divina proportione*, Paganinus De Paganinis, Venice, 1509.
[6] Something is clearly missing in the transmission of knowledges by antiquity on the important concept of gnomon, core of the Euclid Elements.
Leonardo Pisano (Fibonacci) showed to know the formula **(1)** in *Practica Geometriae, II,* 195, in Boncompagni, *Scritti di Leonardo Pisano matematico del secolo decimoterzo*, Tipografia delle scienze matematiche e fisiche, Rome, 1857.
Herz-Fishler noted this formula and its unknown origin, in Herz-Fischler, *A mathematical history of the golden number,* Dover, 1998.

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